2-5 Logarithms: Equations and Other Bases

In the previous section you learned that a logarithm is an exponent of 10. In this section you will learn that it is possible to find logarithms using other positive numbers as the base. Then you will learn how to use the properties of logarithms to solve an equation for an unknown exponent or to solve an equation involving logarithms.

Objective

Use logarithms with base 10 or other bases to solve exponential or logarithmic equations.

Logarithms with Any Base: The Change-of-Base Property

If $x = 10^y$, then y is the base-10 logarithm of x. Similarly, if $x = 2^y$, then y is the base-2 logarithm of x. The only difference is the number that is the base. To distinguish among logarithms with different bases, the base is written as a subscript after the abbreviation *log*. For instance,

$$3 = \log_2 8 \Leftrightarrow 2^3 = 8$$

$$4 = \log_3 81 \Leftrightarrow 3^4 = 81$$

$$2 = \log_{10} 100 \Leftrightarrow 10^2 = 100$$

The symbol $\log_2 8$ is pronounced "log to the base 2 of 8." The symbol $\log_{10} 100$ is, of course, equivalent to log 100, as defined in the previous section. Note that in all cases *a logarithm is an exponent.*

DEFINITION: Logarithm with Any Base

Algebraically:

$\log_b x = y$ if and only if $b^y = x$, where $b > 0$, $b \neq 1$, and $x > 0$

Verbally:

$\log_b x = y$ means that y is the exponent of b that gives x as the answer.

The way you pronounce the symbol for logarithm gives you a way to remember the definition. Examples 1 and 2 show you how to do this.